ON THE MOTION OF NONLINEAR GYROSCOPIC SYSTEMS

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Estimates are presented in this paper, enabling us to decide in certain cases whether the equations of motion arising from the applied (precessional) theory of gyroscopes are applicable for large variations of the position coordinates and of the velocity.

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1. In a general case, the dependence on time of the motion of the platform, of the masses of the gyroscopic system, of the generalized, ordinary and reactional forces, all of which correspond to the cyclic coordinates, is arbitrary and the equations of motion of mechanical systems with gyroscopes are written [1] as

$$\frac{Д}{Дt}\frac{DR}{D\dot{q}_i} - \frac{DR}{Dq_i} = Q_i + \Psi_i \qquad (1.1)$$

Here we use the notation for the derivatives the way it is used in the mechanics of variable masses, where the derivatives are calculated keeping the masses constant; $q_i (i = 1, \ldots, s)$ are the position coordinates; Q_i and Ψ_i are, respectively, the ordinary and the reaction forces of the absolute motion which are functions of their position coordinates, of their time derivatives, and of time; $Д/Дt$ is the partial derivative with respect to time, when m and t are independent variables; D/Dq_i and $D/D\dot{q}_i$ are partial derivatives with respect to the indicated variables when m, t, q_i and $\dot{q}_i$ are independent variables.

The Routh function for the absolute motion of the system, correct within an additive constant, is [1]

$$R = \sum_{k=1}^{r} C_k (H + h_k)\left(\sum_{j=1}^{s} a_j{}^k \dot{q}_j + a_0{}^k\right) + T^* \qquad (H \gg h_k) \qquad (1.2)$$

where C_k is the axial moment of inertia of the kth gyroscope, r is the number of gyroscopes, H is a sufficiently large constant, h_k is a function of time, a_j^k is the cosine of the angle between the angular velocity vector q_j and the axis of the kth gyroscope, a_0^k is the component of the angular velocity of the platform along the axis of the kth gyroscope.

In Formula (1.2) the quantity T^* represents the kinetic energy of the absolute motion of the elements of the suspension of the gyroscopic system, of the inner rings (cases), of the motors of the gyroscopes, and also the kinetic energy of the rotors spinning about their axes.

Substituting (1.2) in Equations (1.1) we obtain

$$\sum_{j=1}^{s} a_{ij}\ddot{q}_j + H\left(\sum_{j=1}^{s} g_{ij}\dot{q}_j + g_{i0}\right) + \Omega_i = 0 \tag{1.3}$$

Here we introduce

$$g_{ij} = \sum_{k=1}^{r} C_k\left(\frac{\partial a_i^k}{\partial q_j} - \frac{\partial a_j^k}{\partial q_i}\right), \qquad g_{i0} = \sum_{k=1}^{r} C_k\left(\frac{\partial a_i^k}{\partial t} - \frac{\partial a_0^k}{\partial q_i}\right)$$

$$\Omega_i = \Phi_i - \Theta_i, \qquad \Phi_i = \frac{\text{Д}}{\text{Д}t}\frac{DT^*}{D\dot{q}_i} - \frac{DT^*}{Dq_i} - \sum_{j=1}^{s} a_{ij}\ddot{q}_j, \qquad a_{ij} = \frac{\partial^2 T^*}{\partial\dot{q}_i\partial\dot{q}_j} \tag{1.4}$$

$$\Theta_i = Q_i + \Psi_i - \sum_{k=1}^{r} C_k h_k\left[\left(\frac{\partial a_i^k}{\partial q_j} - \frac{\partial a_j^k}{\partial q_i}\right)\dot{q}_j + \left(\frac{\partial a_i^k}{\partial t} - \frac{\partial a_0^k}{\partial q_i}\right)\right]$$

Writing down the equations of motion arising from the applied theory of gyroscopes we set $T^* = 0$. Denoting in this case the position coordinates as q_i, we obtain the equations

$$H\left(\sum_{j=1}^{s} g_{ij}g_j + g_{i0}\right) - \Theta_i = 0 \tag{1.5}$$

where among the g_{ij}'s the g_{i0}'s are the position coordinates, and in the term θ_i the position coordinates and the velocities are replaced by g_i's and $\dot{g}_i$'s.

When a gyroscopic system rests on a fixed platform, then $\partial a_i^k/\partial t = a_0^k = 0$, hence $g_{i0} = 0$.

In practice, gyroscopic systems rest on moving platforms and g_{i0} are of the same order as the angular velocity of the earth's rotation, that is, sufficiently small. In some gyroscopic systems the pendular moment appearing in Q_i approaches in magnitude H. We must take therefore into account the peculiar features of the gyroscopic system under investigation.

We shall assume that the matrix $\| g_{ij} \|$ is non-singular, that the coefficients a_{ij}, g_{ij}, and g_{i0} are of the order zero or less (like H), and that all the functions in (1.4) can be expanded in power series.

Let us consider the differential equations containing the small parameter

$$\sum_{j=1}^{s} g_{ij}^{(1)} \dot{q}_{j1} + g_{i0}^{(1)} + \lambda \Phi_i^{(1)} - H^{-1} \Theta_i^{(1)} = 0$$

The superscript (1) indicates that in the corresponding functions the position coordinates and the velocities are replaced by the variables q_{i1} and $\dot{q}_{i1}$.

By the Poincaré small-parameter method [2] the variables q_{i1} and $\dot{q}_{i1}$ are determined from Equation (1.5), correct within the terms of order λ. Assuming that in a special case $\lambda = H^{-1}$, we have

$$\{ q_{i1} - g_i, \ \dot{q}_{i1} - \dot{g}_i \} = O(H^{-1}) \qquad (1\ 6)$$

2. Let the gyroscopic system rest on a fixed platform, and let the generalized forces of the absolute motion contain only terms of zero order with respect to H.

We shall try now to obtain a solution of Equations (1.3) in the form

$$q_i = q_{i1}(H^{-1}t) + x_i(Ht) \qquad (2.1)$$

In our case, the variables q_{i1} satisfy the equations

$$H \sum_{j=1}^{s} g_{ij}^{(1)} \dot{q}_{j1} + \Omega_i^{(1)} = 0 \qquad (2.2)$$

The initial conditions are

$$q_{i1}^{\circ} = q_i^{\circ}, \qquad x_i^{\circ} = 0, \qquad H \sum_{j=1}^{s} g_{ij}^{(1)\circ} \dot{q}_{j1}^{\circ} + \Omega_i^{(1)\circ} = 0, \qquad \dot{x}_i^{\circ} = \dot{q}_i^{\circ} - \dot{q}_{i1}^{\circ} \qquad (2.3)$$

The derivatives with respect to $\tau_1 = H^{-1}t$ and $\tau_2 = Ht$ will be denoted by primes. When $\partial g_{ij}/\partial t$ and $\partial \Omega_i/\partial t$ equal zero, or if they are of the order H^{-1} or less, then we shall say that our gyroscopic system satisfies conditions (A). From Equations (2.2) and conditions (2.3) it follows that q_{i1} and q'_{i1} are of the order zero with respect to H, and further that q''_{i1} are of the order H, and when the conditions (A) are satisfied they are of the order zero with respect to H.

By (1.3), (2.1), (2.2), and by setting $g_{i0} = 0$, we obtain equations which determine x_i:

$$\sum_{j=1}^{s}\left[a_{ij}(q_{m1}(H^{-2}\tau_2)+x_m,\ H^{-1}\tau_2)x_j''+g_{ij}(q_{m1}(H^{-2}\tau_2)+x_m,\ H^{-1}\tau_2)x_j'\right]+$$

$$+H^{-2}\left[\Omega_i(q_{m1}(H^{-2}\tau_2)+x_m,\ H^{-1}q_{m1}'(H^{-2}\tau_2)+Hx_m',\ H^{-1}\tau_2)-\right.$$

$$\left.-\Omega_i(q_{m1}(H^{-2}\tau_2),\ H^{-1}q_{m1}'(H^{-2}\tau_2),\ H^{-1}\tau_2)\right]+H^{-2}f_i(H^{-1},\ x_m,\ \tau_2)=0$$

$$f_i=\sum_{j=1}^{s}a_{ij}\left[q_{m1}(H^{-2}\tau_2)+x_m,\ H^{-1}\tau_2\right]q_{j1}''(H^{-2}\tau_2)+$$

$$+\sum_{j=1}^{s}\left[g_{ij}(q_{m1}(H^{-2}\tau_2)+x_m,\ H^{-1}\tau_2)-g_{ij}(q_{m1}(H^{-2}\tau_2),\ H^{-1}\tau_2)\right]q_{j1}'(H^{-2}\tau_2)$$

When the quantity ξ_i is of the zeroth order, let

$$O\{\Omega_i(\xi_j,\ \eta_j,\ t)\}=\sigma O(\eta_j)$$

Interesting cases in practice are those for which $\sigma = 0,\ 1,\ 2$; we are going to dwell on these cases. The functions f_i are of the order H, and when the conditions (A) are satisfied they are of the order zero with respect to H.

By the Poincaré small-parameter method [2] we obtain

$$\{x_i-y_i,\ x_i'-y_i'\}=O(H^{-1}) \tag{2.4}$$

The variables y_i have the same initial values as the variables x_i, and they satisfy the equations

$$\begin{aligned}\sum_{j=1}^{s}\left[a_{ij}(q_m^\circ+y_m,\ 0)y_j''+g_{ij}(q_m^\circ+y_m,\ 0)y_j'\right]\\ =-H^{-2}\left[\Omega_i(q_m^\circ+y_m,\ Hy_m',\ 0)-\Omega_i(q_m^\circ,\ 0,\ 0)\right]\end{aligned} \tag{2.5}$$

When $\sigma = 0,\ 1$, then the right-hand side of (2.5) can be replaced by zero. When $\sigma = 2$, then a_{ij}, g_{ij}, and the right-hand side of (2.5) is of order zero with respect to H, and by (2.5) the order of y_j and y_j' is the same as the order of y_j° and $y_j'^\circ$, that is, H^{-1}.

Therefore, as a consequence of (1.6), (2.1) and (2.4), we obtain

$$\{q_i-g_i\}=O(H^{-1}) \tag{2.6}$$

In our case Formula (2.6) constitutes the foundation of the applied theory of gyroscopes.

Note. In order to prove the convergence of the series by the Poincaré method, we require that the variables $q_{i1} - g_i$, $\dot{q}_{i1} - \dot{g}_i$ considered in Section 1, and the variables $x_i - y_i$, $x_i' - y_j'$ considered in Section 2, be bounded. Thus, the derived asymptotic estimates are valid for an infinite interval of time only when the above-mentioned variables are bounded. In a general case with a possible instability of the gyroscopic system, the estimates would be valid at any instant of time if the variables were within a domain which, however large, is bounded. This remark applies to everything which follows.

We shall derive estimates for positional velocities. Introducing the parameter $\mu = H^{-2}$, and taking into account that in the general case q_{j1}'' are of the order H, we obtain

$$\{x_i - z_i,\ x_i' - z_i'\} = O(H^{-2}), \qquad \{\dot{x}_i - \dot{z}_i\} = O(H^{-1}) \tag{2.7}$$

The variables z_i have the same initial values as x_i and are determined from the equations

$$\sum_{j=1}^{s} \left[a_{ij}(q_m^\circ + z_m, t)\ddot{z}_j + Hg_{ij}(q_m^\circ + z_m, t)\dot{z}_j\right] = \tag{2.8}$$

$$= -\Omega_i(q_{m1} + z_m,\ H^{-1}q_{m1}' + Hz_m',\ t) + \Omega_i(q_{m1},\ H^{-1}q_{m1}',\ t) -$$

$$- \sum_{j=1}^{s} a_{ij}(q_m^\circ + z_m,\ t)\ddot{q}_{j1}(0)$$

When the (A) conditions are satisfied, then q_{j1}'' are of zero order with respect to H, and on the right-hand side of (2.8) the last term containing $\ddot{q}_{ij}(0)$ can be omitted. If, besides, $\sigma = 0$, then the whole right-hand side of (2.8) can be omitted and set equal zero.

Since by (2.2) the derivatives $\dot{q}_{i1}$ are of the order H^{-1}, on the strength of (1.6), (2.1) and (2.7) we have the following estimate of positional velocities: $\{\dot{q}_i - \dot{z}_i\} = O(H^{-1})$.

We shall examine, for example, a gyroscope of variable mass, whose exact equations of motion are [3]

$$A\ddot{\theta} - A\dot{\psi}^2 \sin\theta\cos\theta + C(\dot{\varphi} + \dot{\psi}\cos\theta)\dot{\psi}\sin\theta = \mathrm{mgl}\sin\theta$$

$$A\frac{d}{dt}(\dot{\psi}\sin^2\theta) + C\frac{d}{dt}[(\dot{\varphi} + \dot{\psi}\cos\theta)\cos\theta] = K\cos\theta$$

$$C\frac{d}{dt}(\dot{\varphi} + \dot{\psi}\cos\theta) = K$$

Here the moments of inertia A and C, the pendular moment mgl, and the reactive moment K are known functions of time; we find that

$$\dot{\varphi} + \dot{\psi}\cos\theta = H + h, \qquad h = \int_0^t \frac{K}{C}\,dt$$

Equations (1.3) assume the form

$$A\ddot{\theta} - A\dot{\psi}^2\sin\theta\cos\theta + C(H+h)\dot{\psi}\sin\theta = \mathrm{mgl}\sin\theta$$

$$A\ddot{\psi}\sin^2\theta + 2A\dot{\theta}\dot{\psi}\sin\theta\cos\theta - C(H+h)\dot{\theta}\sin\theta = 0$$

From the applied theory of gyroscopes we obtain

$$C(H+h)\dot{\psi} = \mathrm{mgl}, \quad \dot{\theta} = 0, \quad \text{or} \quad \theta = \theta_0, \quad \psi = \int_0^t \frac{\mathrm{mgl}}{C(H+h)}\,dt + \psi_0 \tag{2.9}$$

As has been shown, the slow precession (2.9) determines the solution of the exact equations, correct within terms of the order H^{-1}.

We have thus reached the following conclusion: if a fast-spinning gyroscope of variable mass is deflected from its vertical position through an angle θ_0, then it will begin to oscillate with respect to the motion (2.9) in variables θ and ψ, and the amplitude of these oscillations will be of the order H^{-1} and the frequency of the order H.

3. We shall consider now a gyroscopic system on a moving platform with generalized forces of the order H (the pendular moment, for example.) Let $\dot{g}_i$ be determined by differential equations (1.5), and let them be of the order λ with respect to H. The solution of Equations (1.3) will be in the form (2.1). The variables q_{i1} will satisfy the equations

$$H\left(\sum_{j=1}^{s} g_{ij}^{(1)}\dot{q}_{j1} + g_{i0}^{(1)}\right) + \Omega_i^{(1)} = 0 \tag{3.1}$$

In this case we cannot regard q_{i1} as functions of the argument $\tau_i = H^{-1}t$. The equations which determine x_i are

$$\sum_{j=1}^{s}\left[a_{ij}(q_{m1}+x_m,\ H^{-1}\tau_2)x_j'' + g_{ij}(q_{m1}+x_m,\ H^{-1}\tau_2)x_j'\right] +$$
$$+ H^{-2}\left[\Omega_i(q_{m1}+x_m,\ \dot{q}_{m1}+Hx_m',\ H^{-1}\tau_2) - \Omega_i(q_{m1},\ \dot{q}_{m1},\ H^{-1}\tau_2)\right] +$$
$$+ F_i(H^{-1},\ \mu,\ x_m,\ \tau_2) = 0$$

where

$$F_i = \sum_{j=1}^{s} a_{ij}(q_{m1}+x_m,\ H^{-1}\tau_2)\ddot{q}_{j1} + H^{-1}\sum_{j=1}^{s}\left[g_{ij}(q_{m1}+x_m,\ H^{-1}\tau_2) -\right.$$
$$\left.- g_{ij}(q_{m1},\ H^{-1}\tau_2)\right]\dot{q}_{j1} + H^{-1}\left[g_{i0}(q_{m1}+x_m,\ H^{-1}\tau_2) - g_{i0}(q_{m1},\ H^{-1}\tau_2)\right]$$

When ξ_j are of the zero order, then we consider

$$O[\Omega_i(\xi_j,\ \eta_j,\ t)] \leqslant O'', \qquad O'' = \max\{H^2,\ \sigma O(\eta)\} \qquad (\sigma = 0, 1, 2)$$

This corresponds to cases interesting in practice.

The functions F_i depend on two small parameters H^{-1} and μ, and they vanish when the parameters equal zero. Denoting by O' the largest order of H^{-1} and of μ and applying the Poincaré small-parameter method, we obtain

$$O\{x_i - y_i,\ x_i' - y_i'\} = O' \tag{3.2}$$

In this case the variables y_i are determined from the equations

$$\sum_{j=1}^{s} \left[a_{ij}(q_{m1} + y_m,\ H^{-1}\tau_2)\, y_j'' + g_{ij}(q_{m1} + y_m,\ H^{-1}\tau_2)\, y_j'\right] + \tag{3.3}$$
$$+ H^{-2}\left[\Omega_i(q_{m1} + y_m,\ \dot{q}_{m1} + Hy_m',\ H^{-1}\tau_2) - \Omega_i(q_{m1},\ \dot{q}_{m1},\ H^{-1}\tau_2)\right] = 0$$

From Equations (3.3) it follows that y_i and y_i' are of the same order as their initial values, that is, H^{-1}. From the estimates (1.6) and (3.2) we find

$$O\{q_i - g_i\} = O' \tag{3.4}$$

Fromula (3.4) can serve as a criterion in deciding whether the equations of the applied theory of gyroscopes can be used with a moving-platform problem or not.

Consider the example of a gyroscope of constant mass with the axis of the outer gimbal ring fixed in the platform moving with constant speed in constant direction on the earth's surface.

Using the applied theory of gyroscopes ($T^* = 0$) and neglecting the eastern velocity component, we obtain

$$R = C(H + h)(\dot{\psi}\cos\theta - v_N R^{-1}\sin\theta\sin\psi - \omega\cos\varphi\sin\theta\cos\psi + \omega\sin\varphi\cos\theta)$$

Here θ and ψ are the Eulerian angles measured from the local vertical and parallel of latitude (in an easterly direction), v_N is the northern velocity component of the platform, R and ω are, respectively, the radius and the angular velocity of the earth, ϕ is the geocentric latitude. The equations of motion in variables g are

$$\dot{\psi} = -(v_N R^{-1}\cot\theta\sin\psi + \omega\cos\varphi\cot\theta\cos\psi + \omega\sin\varphi) + \frac{\mathrm{mgl}}{C(H + h)}$$
$$\dot{\theta} = v_N R^{-1}\cos\psi - \omega\cos\varphi\sin\psi \tag{3.5}$$

It has been shown previously that the solution of Equations (3.5) represents the motion of a gyroscope in coordinates θ and ψ, correct within terms of order O', where

$$O' = \max\left\{H^{-1}, O\left(\frac{v_N}{R}\right)^2, \; O(\omega^2), \; O\left(\frac{v_N\omega}{R}\right), \; O\left[\frac{v_N \,\mathrm{mgl}}{CR(H+h)}\right], \; O\left[\frac{\omega \,\mathrm{mgl}}{C(H+h)}\right]\right\}$$

4. We shall consider now a gyroscopic system with n auxiliary equations solved for the highest derivatives

$$\ddot{q}_\alpha + \Omega_\alpha(q_i, \dot{q}_i, q_\nu, \dot{q}_\nu, q_\gamma, t) = 0, \quad \dot{q}_\beta + \Omega_\beta(q_i, \dot{q}_i, q_\nu, \dot{q}_\nu, q_\gamma, t) = 0 \quad (4.1)$$

Here $\alpha,\nu = s+1, \ldots, s+n_1$; $\beta,\gamma = s+n_1+1, \ldots, s+n$. The functions Ω_i which appear in Equations (1.3) depend in this case on the same variable as Ω_α, Ω_β; the coefficients a_{ij}, g_{ij}, g_{i0} depend on $s+n$ Lagrangian coordinates.

The equations of motion derived from the applied theory of gyroscopes form a compatible system (1.5) and (4.1) in which the variables q and their derivatives should be replaced by the variables g and their derivatives with the appropriate indices. Further

$$g_\nu^\circ = q_\nu^\circ, \qquad g_\gamma^\circ = q_\gamma^\circ, \qquad \dot{g}_\nu^\circ = \dot{q}_\nu^\circ$$

The replacement of the variables q_{i1}, $q_{\nu 1}$, $q_{\gamma 1}$, $\dot{q}_{i1}$, $\dot{q}_{\nu 1}$ by the variables g_i, g_ν, g_γ, $\dot{g}_i$, $\dot{g}_\nu$ is correct within terms of order H^{-1}.

The equations determining the variables $x_p (p = 1, \ldots, s+n)$ are

$$\sum_{j=1}^{s}\left[a_{ij}(q_{p1}+x_p, H^{-1}\tau_2)x_j'' + g_{ij}(q_{p1}+x_p, H^{-1}\tau_2)x_j'\right] +$$
$$+ H^{-2}\left[\Omega_i(q_{p1}+x_p, \dot{q}_{j1}+Hx_j', \dot{q}_\alpha + Hx_\alpha', H^{-1}\tau_2) -\right.$$
$$\left.- \Omega_i(q_{p1}, \dot{q}_{j1}, \dot{q}_{\alpha 1}, H^{-1}\tau_2)\right] + F_i(H^{-1}, \mu, x_p, \tau_2) = 0$$

$$x_\alpha'' + H^{-2}\left[\Omega_\alpha(q_{p1}+x_p, \dot{q}_{i1}+Hx_i', \dot{q}_{\alpha 1}+Hx_\alpha', H^{-1}\tau_2) - \Omega_\alpha(q_{p1}, \dot{q}_{i1}, \dot{q}_{\alpha 1}, H^{-1}\tau_2)\right] = 0$$

$$x_\beta' + H^{-1}\left[\Omega_\beta(q_{p1}+x_p, \dot{q}_{i1}+Hx_i', \dot{q}_{\alpha 1}+Hx_\alpha' \; H^{-1}\tau_2) - \Omega_\beta(q_{p1}, \dot{q}_{i1}, \dot{q}_{\alpha 1}, H^{-1}\tau_2)\right] = 0$$

where

$$F_i = \sum_{j=1}^{s} a_{ij}(q_{p1} + x_p,\ H^{-1}\tau_2)\ddot{q}_{j1} + H^{-1}\Big\{\sum_{j=1}^{s}\big[g_{ij}(q_{p1} + x_p,\ H^{-1}\tau_2) -$$
$$- g_{ij}(q_{p1},\ H^{-1}\tau_2)\big]\dot{q}_{j1} + g_{i0}(q_{p1} + x_p,\ H^{-1}\tau_2) - g_{i0}(q_{p1},\ H^{-1}\tau_2)\Big\}$$

We shall consider that if $O(\xi_p) = 0$ then

$$O\{\Omega_i(\xi_p,\ \eta_j,\ \zeta_\alpha,\ t)\} \leqslant O'', \qquad O'' = \max\{H^2,\ \sigma O(\eta_j),\ \sigma O(\zeta_\alpha)\} \qquad (\sigma = 0, 1, 2)$$
$$O\{\Omega_\nu(\xi_p,\ \eta_j,\ \zeta_\alpha,\ t)\} \leqslant O'', \qquad O\{\Omega_\gamma(\xi_p,\ \eta_j,\ \zeta_\alpha,\ t)\} \leqslant \{H,\ \sigma_1 O(\eta_j),\ \sigma_1 O(\zeta_\alpha)\}$$
$$(\sigma_1 = 0, 1)$$

By the Poincaré small-parameter method we have

$$\{x_p - y_p,\ x_i' - y_i',\ x_\alpha' - y_\alpha'\} = O'$$

The variables y_i satisfy the equation

$$\sum_{j=1}^{s}\big[a_{ij}(q_{p1} + y_p,\ H^{-1}\tau_2)y_j'' + g_{ij}(q_{p1} + y_p,\ H^{-1}\tau_2)y_j'\big] +$$
$$+ H^{-2}\big[\Omega_i(q_{p1} + y_p,\ \dot{q}_{j1} + Hy_j',\ \dot{q}_{\alpha 1} + Hy_\alpha',\ H^{-1}\tau_2) -$$
$$- \Omega_i(q_{p1},\ \dot{q}_{j1},\ \dot{q}_{\alpha 1},\ H^{-1}\tau_2)\big] = 0$$

The variables y_ν and y_γ satisfy the same equations as x_ν and x_γ.

The variables y_p, y_p' are of the same order as their initial values. Hence

$$\{y_p,\ y_p'\} = O(H^{-1})$$

Finally, we obtain, in the form of the following estimates, the criteria for deciding whether or not the equations derived from the applied theory of gyroscopes are acceptable:

$$O\{q_p - g_p\} = O'' \qquad (p = 1, 2, \ldots, s + n)$$

BIBLIOGRAPHY

1. Novoselov, V.S., Dvizhenie stabilizirovannykh giroskopicheskikh sistem na podvizhnom osnovanii (On the motion of stabilized gyroscopic systems on a moving platform). *PMM* Vol. 23, No. 5, 1959.

2. Golubev, V.V., *Lektsii po analiticheskoi teorii differentsialnykh uravnenii (Lectures on the Analytic Theory of Differential Equations)*. GITTL, 1950.

3. Novoselov, V.S., Reguliarnaia pretsessia giroskopa peremennoi massy (On the regular precession of a gyroscope of variable mass). *Izv. Akad. Nauk SSSR, OTN* No. 11, 1958.

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